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Massive spinors and dS/CFT correspondence

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Abstract: Using the map between free massless spinors on d+1 dimensional Minkowski spacetime and free massive spinors on dS_{d+1}, we obtain the boundary term that should be added to the standard Dirac action for spinors in the dS/CFT correspondence. It is shown that this map can be extended only to theories with vertex $(\bar{\psi}\psi)^2$ but arbitrary $d \geq 1$. In the case of scalar field theories such an extension can be made only for $d = 2, 3, 5$ with vertices ϕ^6, ϕ^4 and ϕ^3 respectively.

Keywords: AdS-CFT and dS-CFT Correspondence, Cosmology of Theories beyond the SM.

Contents

1. Introduction

It is known that the correct action for spinors in AdS_{d+1}/CFT_d correspondence is the sum of the standard Dirac action,

$$S = \int_{\Sigma} d^{d+1}x \bar{\psi}(x) \left(\not{D} - m \right) \psi(x) \,, \tag{1.1}$$

and some boundary term [1, 2, 3]. Specially in [1], Henneaux has shown that the boundary term can be determined by the stationary conditions on the solutions of Dirac equation with a definite asymptotic behavior. In those papers, the euclidean AdS space is considered as the domain $t > 0$ with metric,

$$ds^2 = \frac{1}{t^2} \eta^{\mu\nu} dx^{\mu} dx^{\nu} \,, \tag{1.2}$$

where $x^0 = t$ and $\eta^{\mu\nu} = (+, +, \ldots, +)$. It is also shown that the boundary term gives the action for spinors on the boundary with expected conformal weight in AdS/CFT correspondence. Since, dS_{d+1} space can be also described as the domain $t > 0$ with metric (1.2) in which $\eta^{\mu\nu} = (+, -, \ldots, -)$, it is reasonable to consider similar boundary terms in dS/CFT correspondence.

Recently we showed that massless scalar fields on $d+1$ dimensional Minkowski spacetime are dual to scalar fields on dS_{d+1} with mass $m^2 = \frac{d^2-1}{4}$ [4]. In fact, if $\phi(\vec{x}, t)$ is the solution of Klein-Gordon equation in $d+1$ dimensional Minkowski spacetime M_{d+1},

$$\eta^{\mu\nu} \partial_{\mu} \partial_{\nu} \phi(t, \vec{x}) = 0 \,, \tag{1.3}$$

then $\Phi(\vec{x}, t) = t^{\frac{d-1}{2}} \phi(\vec{x}, t)$ is a massive scalar on dS_{d+1}, satisfying the Klein-Gordon equation,

$$\left(t^2 \partial_t^2 + (1-d) t \partial_t - t^2 \nabla^2 + \frac{d^2-1}{4} \right) \Phi(\vec{x}, t) = 0 \,. \tag{1.4}$$

Using this map we obtained the action of the dual CFT on the boundary of dS space given in [7] by inserting the solution of the Klein-Gordon equation in M_{d+1} in terms of the initial data given on the hypersurface $t = 0$ into the action of massless scalars on M_{d+1} and identifying the initial data with the CFT fields. The same result was obtained by mapping massless scalars from euclidean space to euclidean AdS.

In M_{d+1}, solutions of Dirac equation are solutions of the Klein-Gordon equation. Therefore, one can obtain some information about spinors and the dS/CFT correspondence by mapping massless spinors from $d+1$ dimensional Minkowski (euclidean) spacetime to de Sitter (euclidean AdS) space.

As is mentioned in [4], one can use the relation between massless fields in Minkowski (euclidean) spacetime and massive fields in dS (euclidean AdS) space to prove the holographic principle for the domain $t \geq 0$ of Minkowski (euclidean) spacetime in the case of massless fields. In the case of Minkowski spacetime a covariant description of the holographic principle can be given by considering a covariant boundary instead of the hypersurface $t = 0$ which here is the space-like boundary of the domain $t \geq 0$ [5]. Similar ideas are considered in ref. [6] where the relation between massive and massless scalar fields in Minkowski spacetime and on (anti-)de Sitter space, one dimension lower, is studied.

The organization of paper is as follows. In section 2, we give the solution of Dirac equation for massless spinors in the domain $t \geq 0$ of Minkowski spacetime in terms of the initial data given on the boundary $t = 0$. Specially we show that by adding a suitable boundary term to the standard Dirac action for spinors, not only the hermitean condition of the action can be satisfied but one can also obtain the Dirac equation without imposing any condition on fields living on the boundary. Finally we will consider this unique boundary term as the action of CFT on the boundary. In section 3, we show that the most general solution of Dirac equation on dS (euclidean AdS) space (spinors with arbitrary mass) can be given in terms of massless spinors in Minkowski spacetime with the same dimensionality. Finally we show that the map between fields in Minkowski spacetime and dS space can be extended to field theories with vertex $(\bar{\psi}\psi)^2$ but arbitrary $d \geq 1$ in the case of spinors. For scalar field theories this extension is applicable only in $d = 2, 3, 5$ and for theories with vertices ϕ^6, ϕ^4 and ϕ^3 respectively. We close the paper with a brief summary of results. There is also an appendix in which we have given some details about the general solution of Dirac equation for massless spinors in M_{d+1}. These details are considerable since they show the agreement of our results with those of dS/CFT in the case of scalar fields reported in [4, 7]. In addition they make the connection of our results with previous attempts on dS/CFT in the case of spinors [1]–[3].

2. Massless fermions on M_{d+1}

The Dirac equation for massless fermions, $i\gamma^\mu \partial_\mu \psi(t, \vec{x}) = 0$ has solutions,

$$\psi(\vec{x}, t) = \int d^d\vec{k}\tilde{\psi}^\pm(\vec{k}) e^{i\vec{k}.\vec{x}} e^{\pm i\omega t}\,, \tag{2.1}$$

where,

$$\left(\pm\omega\gamma^0 + \gamma^i \vec{k}_i\right) \tilde{\psi}^\pm(\vec{k}) = 0\,, \qquad \omega = \left|\vec{k}\right|. \tag{2.2}$$

γ^0 and γ^i's are Dirac matrices satisfying the anti-commutation relation $\{\gamma^\mu, \gamma^\nu\} = 2\eta^{\mu\nu}$, in which $\eta^{\mu\nu} = (+, -, \ldots, -)$. One can show that $\tilde{\psi}^+ = \pm\gamma^0\tilde{\psi}^-$, therefore a general solution, $\psi(\vec{x}, t)$ of the Dirac equation can be decomposed as $\psi(\vec{x}, t) = \psi^{(1)}(\vec{x}, t) + \psi^{(2)}(\vec{x}, t)$, where

$$\begin{aligned}
\psi^{(1)}(\vec{x}, t) &= \int d^d\vec{k}\left(e^{-i\omega t} + \gamma^0 e^{i\omega t}\right)\tilde{\psi}^{(1)}(\vec{k})e^{i\vec{k}.\vec{x}}\,, \\
\psi^{(2)}(\vec{x}, t) &= \int d^d\vec{k}\left(e^{-i\omega t} - \gamma^0 e^{i\omega t}\right)\tilde{\psi}^{(2)}(\vec{k})e^{i\vec{k}.\vec{x}}\,.
\end{aligned} \tag{2.3}$$

If one decomposes $\tilde{\psi}^{(a)}(\vec{k})$ as $\tilde{\psi}^{(a)} = \tilde{\psi}^{(a)}_+ + \tilde{\psi}^{(a)}_-$, $a = 1, 2$, where ,

$$\tilde{\psi}^{(a)}_+ = \frac{1+\gamma^0}{2}\tilde{\psi}^{(a)}\,, \qquad \tilde{\psi}^{(a)}_- = \frac{1-\gamma^0}{2}\tilde{\psi}^{(a)}\,, \tag{2.4}$$

and using eq. (2.3) one can show that the initial data $\psi_0(\vec{x}) = \psi(\vec{x}, 0)$ only determines $\tilde{\psi}^{(1)}_+$ and $\tilde{\psi}^{(2)}_-$:

$$\tilde{\psi}^{(1)}_+(\vec{k}) = \int d^d\vec{x}e^{-i\vec{k}.\vec{x}}\psi_0^+(\vec{x})\,, \qquad \tilde{\psi}^{(2)}_-(\vec{k}) = \int d^d\vec{x}e^{-i\vec{k}.\vec{x}}\psi_0^-(\vec{x})\,, \tag{2.5}$$

where $\psi_0^\pm(\vec{x}) = \frac{1\pm\gamma^0}{2}\psi_0(\vec{x})$. Defining,

$$\chi^+(\vec{x}) = \int d^d\vec{k}e^{i\vec{k}.\vec{x}}\tilde{\psi}^{(1)}_-(\vec{k})\,, \qquad \chi^-(\vec{x}) = \int d^d\vec{k}e^{i\vec{k}.\vec{x}}\tilde{\psi}^{(2)}_+(\vec{k})\,, \tag{2.6}$$

and using eq. (2.5), one can determine $\psi^{(a)}(\vec{x}, t)$ in terms of fields $\psi_0^\pm(\vec{x})$ and $\chi^\pm(\vec{x})$ living on the hypersurface $t = 0$, as follows:

$$\psi^{(a)}(\vec{x}, t) = \int d^d\vec{k}d^d\vec{y}\left(e^{-i\omega t} \pm \gamma^0 e^{i\omega t}\right)\left(\psi_0^\pm(\vec{y}) + \chi^\pm(\vec{y})\right)e^{i\vec{k}.(\vec{x}-\vec{y})}\,. \tag{2.7}$$

Since $\gamma^0\psi_0^\pm(\vec{x}) = \pm\psi_0^\pm(\vec{x})$, and $\gamma^0\chi^\pm = \mp\chi^\pm$, one can decompose $\psi(\vec{x}, t) = \psi^{(1)}(\vec{x}, t) + \psi^{(2)}(\vec{x}, t)$ as eigenvectors of γ^0, say $\psi_\pm$ ($\gamma^0\psi_\pm = \pm\psi_\pm$), as follows,

$$\begin{aligned}
\psi(\vec{x}, t) &= \psi_+(\vec{x}, t) + \psi_-(\vec{x}, t) \\
\psi_+(\vec{x}, t) &= \int d^d\vec{k}d^d\vec{y}\left[\left(e^{-i\omega t} + \gamma^0 e^{i\omega t}\right)\psi_0^+(\vec{y}) + \left(e^{-i\omega t} - \gamma^0 e^{i\omega t}\right)\chi^-(\vec{y})\right]e^{i\vec{k}.(\vec{x}-\vec{y})} \\
\psi_-(\vec{x}, t) &= \int d^d\vec{k}d^d\vec{y}\left[\left(e^{-i\omega t} - \gamma^0 e^{i\omega t}\right)\psi_0^-(\vec{y}) + \left(e^{-i\omega t} + \gamma^0 e^{i\omega t}\right)\chi^+(\vec{y})\right]e^{i\vec{k}.(\vec{x}-\vec{y})}\,.
\end{aligned} \tag{2.8}$$

As is shown in the appendix, $\chi^\pm$ can be determined in terms of $\psi_0^\pm(\vec{x})$.

Dirac equation can be obtained from the Dirac action for spinors,

$$S = i\int_\Sigma \bar{\psi}\overrightarrow{\not{\partial}}\psi \tag{2.9}$$

if one assume that Dirac fields vanish on the boundary. This assumption is also necessary for lagrangian density $i\bar{\psi}\overrightarrow{\not{\partial}}\psi$ to be hermitean. If one rewrite the action as,

$$S_D = i\int_\Sigma \bar{\psi}\frac{\overleftarrow{\not{\partial}} + \overrightarrow{\not{\partial}}}{2}\psi \tag{2.10}$$

then the lagrangian density is hermitean with no condition on fields on the boundary. In order to obtain Dirac equation, one can also add the following boundary term to the action,

$$S_b = \int_{\partial\Sigma} \bar{\psi}(\vec{x})\gamma^\mu\psi(\vec{x})n_\mu\,, \tag{2.11}$$

in which n_μ is the unit vector perpendicular to boundary. This term cancels the contribution from the variation of fields on the boundary to δS. Therefore $\delta S = 0$ gives the Dirac equation with no preassumption about fields on the boundary.

Now assume that boundary is the hypesurface $t = 0$ and insert $\psi(\vec{x}, t)$ the solution of the Dirac equation into the action $S_D + S_b$. S_D vanishes by construction and,

$$S_b = \int d^d\vec{x}\bar{\psi}(\vec{x})\gamma^0\psi(\vec{x})\,. \tag{2.12}$$

Using the identity,

$$\psi(\vec{x}) = \int d^d\vec{k} \int d^d\vec{y} e^{i\vec{k}.(\vec{x}-\vec{y})}\psi(\vec{y})\,, \tag{2.13}$$

after some calculations, S_b (2.12) can be given as follows,

$$S_b = \text{Const.} \int d^d\vec{y}d^d\vec{z}\bar{\psi}(\vec{y})\gamma^0\psi(\vec{z})F(\vec{y} - \vec{z})\,, \tag{2.14}$$

where $F(\vec{x}) = \int d^d\vec{k}e^{i\vec{k}.\vec{x}}$. $F(\vec{x})$ is simply equal to $\delta^d(\vec{x})$. A representation of the Dirac delta function appropriate for our purposes can be obtained by noting that $F(\vec{x}) = F(\mathbf{R}.\vec{x})$ for any rotation $\mathbf{R} \in SO(d)$ and $F(s\vec{x}) = s^{-d}F(\vec{x})$ for any $s > 0$. Consequently,

$$\vec{x}.\nabla F(\vec{x}) = r\frac{\partial}{\partial r}F(r) = -d\ F(\vec{x})\,,$$

where $r = |\vec{x}|$. Therefore $F(\vec{x}) = \text{Const.}|\vec{x}|^{-d}$. Inserting $F(\vec{y} - \vec{z})$ into S_b, one finally obtains,

$$S_b = \text{Const.} \int d^d\vec{y}d^d\vec{z}\frac{\bar{\psi}(\vec{z})\gamma^0\psi(\vec{y})}{|\vec{y} - \vec{z}|^d} \tag{2.15}$$

3. Spinors on dS_{d+1}

In the description of dS_{d+1} as the domain $t > 0$ with metric $ds^2 = t^{-2}(dt^2 - d\vec{x}^2)$, the Klein-Gordon equation for massive scalar fields is,

$$\left(t^2(\partial_t^2 - \partial_{\vec{x}}^2) + (1-d)t\partial_t + m^2\right)\phi = 0\,. \tag{3.1}$$

If one write the above equation in the following form,

$$\left(t^2(\partial_t^2 - \partial_{\vec{x}}^2) + (1-d)t\partial t + m^2\right)\psi = \left(t\gamma^\mu\partial_\nu + \alpha\gamma^0 + \zeta\right)\left(t\gamma^\nu\partial_\nu + \beta\gamma^0 + \xi\right)\psi\,, \tag{3.2}$$

where ψ is a spinor field, then one verify that

$$\beta = 1 + \alpha = \frac{1-d}{2}, \qquad \xi = \zeta = 0\,, \qquad m^2 = \frac{d^2-1}{4}\,. \tag{3.3}$$

Therefore, for fields with mass $m^2 = (d^2-1)/4$, one can rewrite the Klein-Gordon equation (3.1), as

$$\left(t\gamma^\mu\partial_\nu + \frac{1-d}{2}\gamma^0\right)\psi = 0\,. \tag{3.4}$$

As we observed in eqs. (1.3) and (1.4), massive scalar fields on dS_{d+1} with mass $m^2 = \frac{d^2-1}{4}$ are related to massless fields on M_{d+1}. It is easy to verify that if ψ is a solution of Dirac equation for massless spinors in M_{d+1} then $\Psi = t^{\frac{d-1}{2}}\psi$ is a solutions of eq. (3.4). If one rewrite eq. (3.4) in terms of the Dirac operator in dS_{d+1}, $\not{D} = t\not{\partial} - \frac{d}{2}\gamma^0$, one obtains,

$$\left(\not{D} + \frac{1}{2}\gamma^0\right)\Psi = 0\,. \tag{3.5}$$

Using the general solution of massless spinors in M_{d+1} eq. (2.8) one verifies that $\Psi_\pm = t^{\frac{d-1}{2}}\psi_\pm$ are massive spinors in dS_{d+1} with mass $m_\pm = \mp\frac{1}{2}$ respectively. In general, one can show that the most general solution of Dirac equation for spinors with mass m, $(\not{D}-m)\Psi = 0$, can be given in terms of $\psi_\pm$, given in (2.8), as follows:

$$\Psi = t^{\frac{d}{2}-m}\psi_- + t^{\frac{d}{2}+m}\psi_+\,. \tag{3.6}$$

In words, all massive spinors in dS_{d+1} space can be considered as images of massless spinors in M_{d+1}. The extension of this result to massive spinors in euclidean AdS_{d+1} and massless spinors in $d+1$ dimensional euclidean space is straightforward.

By construction, the hypersurface $t = 0$ in M_{d+1} is isomorphic to the boundary of dS_{d+1}. If one identifies the CFT fields on the boundary of dS_{d+1} with initial data ψ_0 and χ on the hypersurface of M_{d+1}, then one can consider the action S_b given in eq. (2.15) as the action of CFT on the boundary of dS_{d+1}.

The map $\psi_+ \to \Psi_+ = t^{\frac{d}{2}+m}\psi_+$ can be used to find the Minkowski dual of certain fermionic vertices on dS space. Consider a vertex $V(\bar{\Psi}_+\Psi_+) = g(\bar{\Psi}_+\Psi_+)^n$ in dS space where n is some integer to be determined. The Dirac equation is

$$(\not{D} - m)\Psi_+ = g\bar{\Psi}^n\Psi^{n-1}\,, \tag{3.7}$$

in which g is the coupling constant in dS space. Rewriting the above equation in terms of massless spinor ψ_+, one obtains,

$$\not{\partial}\psi_+ = g(t)\bar{\psi}_+^n\psi_+^{n-1}\,, \tag{3.8}$$

where,

$$g(t) = gt^{-(\frac{d}{2}+m+1)}t^{(2n-1)(\frac{d}{2}+m)}\,, \tag{3.9}$$

is the coupling in the Minkowski spacetime. To obtain the above relation we have used the identity, $(\not{D} - m)\Psi_+ = t^{\frac{d}{2}+m+1}\ \not{\partial}\psi_+$. The Requirement that $g(t)$ is constant imposes the following identity:

$$n = 1 + \frac{1}{d+2m}\,, \qquad n \in \mathbb{N}\,. \tag{3.10}$$

Therefore, $m = -\frac{d-1}{2}$ and $n = 2$. One can verify that the field theory with $n = 2$ vertex is renormalizable for $d \leq 1$.

To study the differences between scalars and spinors in the map $M_{d+1} \rightarrow dS_{d+1}$, it is useful to search for those scalar field vertices that could be mapped from Minkowski spacetime to dS space. As we explained in section 1, massless scalar fields in M_{d+1} can be only mapped to massive scalar fields in dS_{d+1} with mass $m^2 = \frac{d^2-1}{4}$. Using the identity,

$$\left(t^2\partial_t^2 + (1-d)t\partial_t - t^2\nabla^2 + \frac{d^2-1}{4}\right)\Phi(\vec{x},t) = t^{\frac{d-1}{2}+2}\left(\partial_t^2 - \nabla^2\right)\phi\,, \tag{3.11}$$

in which $\Phi = t^{\frac{d-1}{2}}\phi$, and considering a vertex $V(\phi) = \phi^{\tilde{n}}$, one can show that

$$\tilde{n} = 2 + \frac{4}{d-1}\,. \tag{3.12}$$

Since $\tilde{n}$ is an integer, the above identity can be satisfied only for special values of d: $d \in \{2,3,5\}$. The corresponding value for $\tilde{n}$ are $6,4,3$. All such theories are renormalizable.

4. Summary

Studying the map between massless fields in $d+1$ dimensional Minkowski spacetime (M_{d+1}) and massive fields on de Sitter space dS_{d+1}, we found that $\psi_\pm$, the massless scalars in M_{d+1} which are eigenvectors of γ^0 with eigenvalues ±1 can be mapped to massive spinors in dS_{d+1} with mass, $m = \mp\frac{1}{2}$ respectively. We found that the conformal weight of dual spinors on the boundary of dS_{d+1} space is $\Delta = \frac{d}{2}$ and the CFT action is

$$S_b = \text{Const.}\int_{\partial\Sigma}\frac{\bar{\psi}(\vec{z})\gamma^0\psi(\vec{y})}{|\vec{y}-\vec{z}|^d}\,, \tag{4.1}$$

which is in agreement with the action of scalar fields obtained in dS/CFT correspondence. We observed that massive spinors on dS_{d+1} with arbitrary mass can be given in terms of $\psi_\pm$ as follows,

$$\Psi = t^{\frac{d}{2}-m}\psi_- + t^{\frac{d}{2}+m}\psi_+\,. \tag{4.2}$$

Using the above identity, we found that the map between free field theories can be extended to theories with vertex $(\bar{\psi}\psi)^2$ for spinors with mass $m = -\frac{d-1}{2}$ for arbitrary $d \geq 1$. In the case of scalar fields similar considerations showed that only for $d = 2,3,5$, the ϕ^6, ϕ^4 and ϕ^3 theories can be mapped from M_{d+1} to dS_{d+1} respectively.

A. Spinors on Minkowski space-time

In this appendix we obtain the field $\chi(\vec{x})$ in terms of $\psi_0(\vec{x})$. We also give the action S_b in terms of ψ_0 and $\dot{\psi}_0$. The notation we use here for the Fourier components of fields differs slightly with those used in the main body of the paper.

From eq. (2.7) one can show that,

$$\begin{aligned} i\dot{\psi}^\pm(\vec{x},0) &= \int d^d\vec{y}d^d\vec{k}\ \omega\chi^\pm(\vec{y})e^{i\vec{k}.(\vec{x}-\vec{y})} \\ &= \int d^d\vec{k}\ \omega\tilde{\chi}^\pm(\vec{k})e^{i\vec{k}.\vec{x}}\,. \end{aligned} \tag{A.1}$$

On the other hand, $i\dot{\psi} = -i\gamma^0\gamma^i\partial_i\psi$. Thus, using the identity $\psi_0^{\pm}(\vec{x}) = \int d^d\vec{k}e^{i\vec{k}.\vec{x}}\tilde{\psi}_0^{\pm}(\vec{k})$, one verifies that

$$\int d^d\vec{k}\ \omega\tilde{\chi}^{\pm}(\vec{k})e^{i\vec{k}.\vec{x}} = \int d^d\vec{y}d^d\vec{k}\ \gamma^0\gamma^i k_i e^{i\vec{k}.\vec{x}}\tilde{\psi}_0^{\pm}(\vec{k})\,, \tag{A.2}$$

which results in the following identity:

$$\begin{aligned} \tilde{\chi}^{\pm}(\vec{k}) &= \gamma^0\gamma^i\frac{k_i}{\omega}\tilde{\psi}^{\pm}(\vec{k}) \\ &= \mp\gamma^i\frac{k_i}{\omega}\tilde{\psi}^{\pm}(\vec{k})\,. \end{aligned} \tag{A.3}$$

Using the above identity one can also show that

$$\tilde{\psi}^{\pm}(\vec{k}) = \mp\gamma^i\frac{k_i}{\omega}\tilde{\chi}^{\pm}(\vec{k})\,. \tag{A.4}$$

To action S_b can be given in terms of ψ_0 and $\dot{\psi}_0$ as follows (see eq. (2.12)):

$$\begin{aligned} S_b &= \int d^d\vec{x}\bar{\psi}(\vec{x})\gamma^0\psi(\vec{x}) \\ &= \int d^d\vec{x}\left(\bar{\psi}^+(\vec{x})\psi^+(\vec{x}) - \bar{\psi}^-(\vec{x})\psi^-(\vec{x})\right) \\ &= \int d^d\vec{k}\left(\tilde{\bar{\psi}}^+(\vec{k})\tilde{\psi}^+(\vec{k}) - \tilde{\bar{\psi}}^-(\vec{k})\psi^-(-\vec{k})\right) \\ &= \int d^d\vec{k}\ \tilde{\bar{\psi}}^+(\vec{k})\tilde{\psi}^+(-\vec{k}) + \int d^d\vec{k}\ \tilde{\bar{\chi}}^-(\vec{k})\tilde{\chi}^-(-\vec{k})\,, \end{aligned} \tag{A.5}$$

where to obtain the last equality we have used eq. (A.4). Using the Fourier transformation one obtains,

$$\begin{aligned} \int d^d\vec{k}\ \tilde{\bar{\psi}}^+(\vec{k})\tilde{\psi}^+(-\vec{k}) &= \int d^d\vec{y}d^d\vec{z}\bar{\psi}^+(\vec{y})\psi^+(\vec{z})\int d^d\vec{k}\ e^{i\vec{k}.(\vec{y}-\vec{z})} \\ &= \text{Const.}\int d^d\vec{y}d^d\vec{z}\frac{\bar{\psi}^+(\vec{y})\psi^+(\vec{z})}{|\vec{y}-\vec{z}|^d}\,. \end{aligned} \tag{A.6}$$

The last equality can be obtained by considering the variation of the integrand under rescaling and rotation.

Furthermore,

$$\begin{aligned} \int d^d\vec{k}\ \tilde{\bar{\chi}}^-(\vec{k})\tilde{\chi}^-(-\vec{k}) &= \int d^d\vec{k}\ \frac{1}{\omega^2}\tilde{\dot{\bar{\psi}}}^-(\vec{k})\tilde{\dot{\bar{\psi}}}^-(-\vec{k}) \\ &= \int d^d\vec{y}d^d\vec{z}\dot{\bar{\psi}}^-(\vec{y})\dot{\psi}^-(\vec{z})\int d^d\vec{k}\ \frac{1}{\omega^2}e^{i\vec{k}.(\vec{y}-\vec{z})} \\ &= \text{Const.}\int d^d\vec{y}d^d\vec{z}\frac{\dot{\bar{\psi}}^-(\vec{y})\dot{\psi}^-(\vec{z})}{|\vec{y}-\vec{z}|^{d-2}}\,. \end{aligned} \tag{A.7}$$

This result is in agreement with those obtained in refs. [4, 7] for scalar fields.

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